

Linear D.E. of higher order with constant coefficients.

$$\textcircled{1} \left\{ \begin{array}{l} a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y^{(1)} + a_n y = v(x) \\ \text{or} \\ (a_0 D^n + a_1 D^{n-1} + \dots + a_{n-1} D + a_n) y = v(x) \\ \text{or} \\ F(D) y = v(x) \end{array} \right. \quad y^{(n)} = \frac{d^n y}{dx^n}$$

Sol of $\textcircled{1}$ $v(x) = 0$ Homogeneous
 $v(x) \neq 0$ Non-Homog.

$$D = \frac{d}{dx}$$

$$D^2 = \frac{d^2}{dx^2}$$

Sol of $\textcircled{1}$ $y = y_c + y_p = \text{C.F.} + \text{P.I.}$

Complementary function Particular Integral

Complementary fⁿ: The sol. of $F(D)y = 0$

Particular Integral: $y_p = \frac{1}{F(D)} v(x)$

How to find C.F.:

1) Write the given D.E. in the form of operator D.

$$(a_0 D^n + a_1 D^{n-1} + \dots + a_{n-1} D + a_n) y = 0$$

2) Write Auxiliary eqⁿ in m. $D \rightarrow m$

$$a_0 m^n + a_1 m^{n-1} + \dots + a_{n-1} m + a_n = 0$$

This is a polynomial of degree n. This has n roots.

Case I: When all roots are distinct.

$\lambda_1, \lambda_2, \dots, \lambda_n$

$$y_c = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + \dots + C_n e^{\lambda_n x}$$

Case II: When few roots are repeated.

i) $\lambda_1 = \lambda_2 = \lambda_3$ & $\lambda_4, \lambda_5, \dots, \lambda_n$ are distinct

$$y_c = (C_1 + C_2 x + C_3 x^2) e^{\lambda_1 x} + C_4 e^{\lambda_4 x} + C_5 e^{\lambda_5 x} + \dots + C_n e^{\lambda_n x}$$

$$y_c = (C_1 + C_2 x + C_3 x^2) e^{\lambda_1 x} + C_4 e^{\lambda_4 x} + C_5 e^{\lambda_5 x} + \dots + C_n e^{\lambda_n x}$$

ii) $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \dots, \lambda_n$ are distinct

$$y_c = (C_1 + C_2 x) e^{\lambda_1 x} + C_3 e^{\lambda_3 x} + C_4 e^{\lambda_4 x} + \dots + C_n e^{\lambda_n x}$$

Case III When roots are complex $\alpha \pm i\beta$

$$y_c = e^{\alpha x} (C_1 \cos \beta x + C_2 \sin \beta x)$$

Case IV Complex roots are repeated. $\alpha \pm i\beta$ & $\alpha \pm i\beta$

$$y_c = e^{\alpha x} [(C_1 + C_2 x) \cos \beta x + (C_3 + C_4 x) \sin \beta x]$$

Case V Roots are surds $\alpha \pm \sqrt{\beta}$ 2 & 53

$$y_c = e^{\alpha x} [C_1 \cosh \sqrt{\beta} x + C_2 \sinh \sqrt{\beta} x]$$

Q.1. $\frac{d^3 y}{dx^3} - 13 \frac{dy}{dx} + 12y = 0$

$$(D^3 - 13D + 12)y = 0$$

A.E. $m^3 - 13m + 12 = 0$

$$(m-1)(m^2 + m - 12) = 0$$

$$(m-1)(m^2 + 4m - 3m - 12) = 0$$

$$(m-1)[m(m+4) - 3(m+4)] = 0$$

$$(m-1)(m-3)(m+4) = 0$$

$$m = 1, 3, -4$$

$$y_c = C_1 e^x + C_2 e^{3x} + C_3 e^{-4x}$$

Q.2. $\frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 4y = 0$

$$(D^3 - 3D^2 + 4)y = 0$$

$$m^3 - 3m^2 + 4 = 0$$

A.E. $(m+1)(m^2 - 4m + 4) = 0$

$$\begin{array}{r} m^2 - 4m + 4 \\ m+1 \overline{) m^3 - 3m^2 + 4} \end{array}$$

A.E. $m^2 - 3m + 4 = 0$
 $(m+1)(m^2 - 4m + 4) = 0$
 $(m+1)(m-2)^2 = 0$

$m = -1, 2, 2$

$y_c = C_1 e^{-x} + (C_2 + C_3 x) e^{2x}$

Q. 3. $(D^4 + D^2 + 1)y = 0$

A.E. $m^4 + m^2 + 1 = 0$

$m^4 + 2m^2 + 1 - m^2 = 0$

$(m^2 + 1)^2 - m^2 = 0$

$(m^2 - m + 1)(m^2 + m + 1) = 0$

$m_1 = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$m_2 = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$y_c = e^{x/2} \left(C_1 \cos \frac{\sqrt{3}}{2} x + C_2 \sin \frac{\sqrt{3}}{2} x \right) + e^{-x/2} \left(C_3 \cos \frac{\sqrt{3}}{2} x + C_4 \sin \frac{\sqrt{3}}{2} x \right)$

Q. 4. $(D^4 - 1)y = 0$